Risk-Based Sampling: A Summary of Findings

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Risk-Based Sampling

- Powell, M. 2014. Optimal Food Safety Sampling Under a Budget Constraint. Risk Analysis. 34(1): 93-100.
- Powell, M. 2015. Risk-Based Sampling: I Don’t Want to Weight in Vain. Risk Analysis. 35(12):2172-2182.

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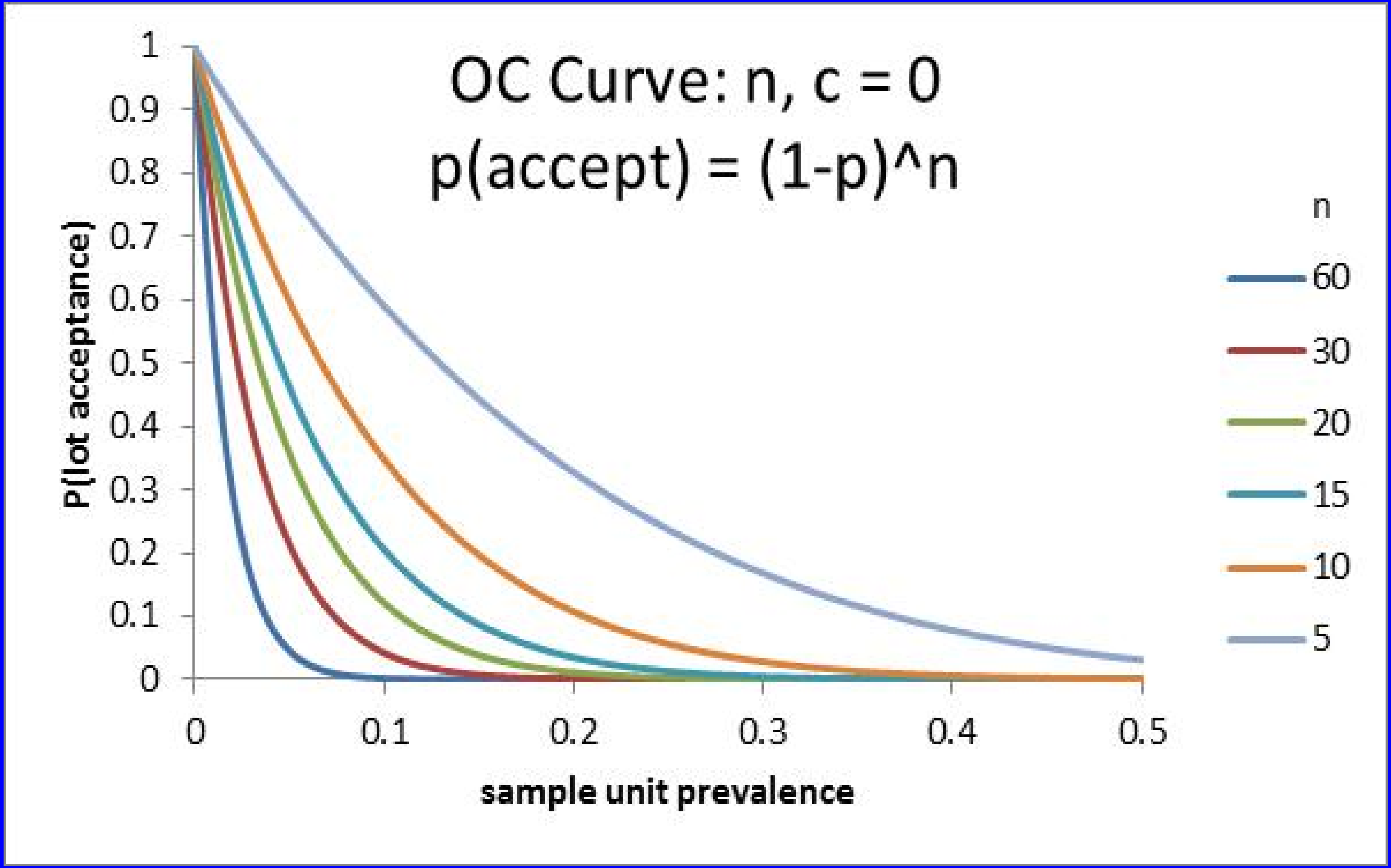
Conventional Lot Acceptance Sampling Plan Design



Simple Optimization Model

- $Max\ L_R = m[1 - q^n]$
  - $L_R =$ contaminated lots rejected
  - $m =$ lots
  - $n =$ samples per lot
  - $q = (1 - \text{p})$
  - $p =$ sample unit prevalence
  - 1-q^n = p(reject lot)
- *S.t.:* Budget constraint (C_T)

Simple Optimization Model

- $Max\ L_R = \frac{C_T}{C_l + nCn}[1 - q^n]$
  - $m = \frac{C_T}{C_l + nC_n}$ (budget constraint)
  - $C_T =$ budgeted total sampling cost ($)
  - $C_l =$ cost per lot ($)
  - $C_n =$ cost per sample ($)

$$n_{opt}(C_T, C_l, C_n, p) \rightarrow \frac{\delta(L_R)}{\delta(n)} = \frac{[C_l + nC_n][-C_T q^n ln(q)] - [C_T(1 - q^n)]C_n}{[C_l + nC_n]^2} = 0$$

Simple Optimization Model

- Obj Fxn: $L_R = m(1 - q^n) = f(m, n|q)$
- Constraint: $C_T \geq m(C_l + nC_n)$
- $L = f(m, n|q) + \lambda[C_T - m(C_l + nC_n)]$

Simple Optimization Model

1) $\frac{\delta L}{\delta m} = \frac{\delta f}{\delta m} - \lambda(C_l + nC_n) = 0$

2) $\frac{\delta L}{\delta n} = \frac{\delta f}{\delta n} - \lambda m C_n = 0$

3) $\frac{\frac{\delta f}{\delta m}}{\frac{\delta f}{\delta n}} = \frac{C_l + nC_n}{mC_n}$

4) $n = \frac{\frac{\delta f}{\delta m}}{\frac{\delta f}{\delta n}} m - \frac{C_l}{C_n}$

5) $\frac{\frac{\delta f}{\delta m}}{\frac{\delta f}{\delta n}} = \frac{(1-q^n)}{-mq^n ln(q)}$

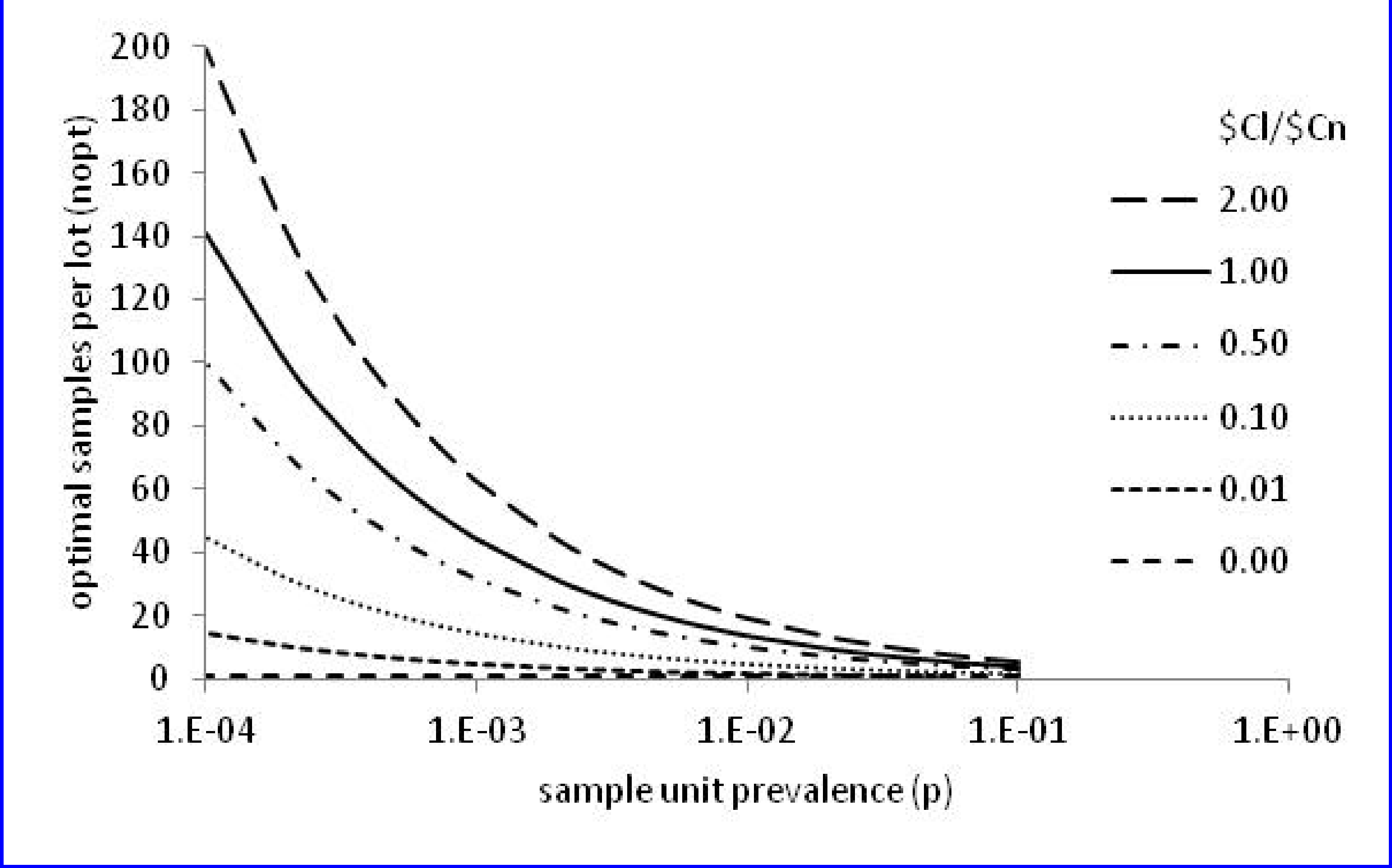
6) $n + \frac{(1-q^n)}{q^n ln(q)} + \frac{C_l}{C_n} = 0$

Note: $n_{opt} = f\left(p, \frac{C_l}{C_n}\right)$

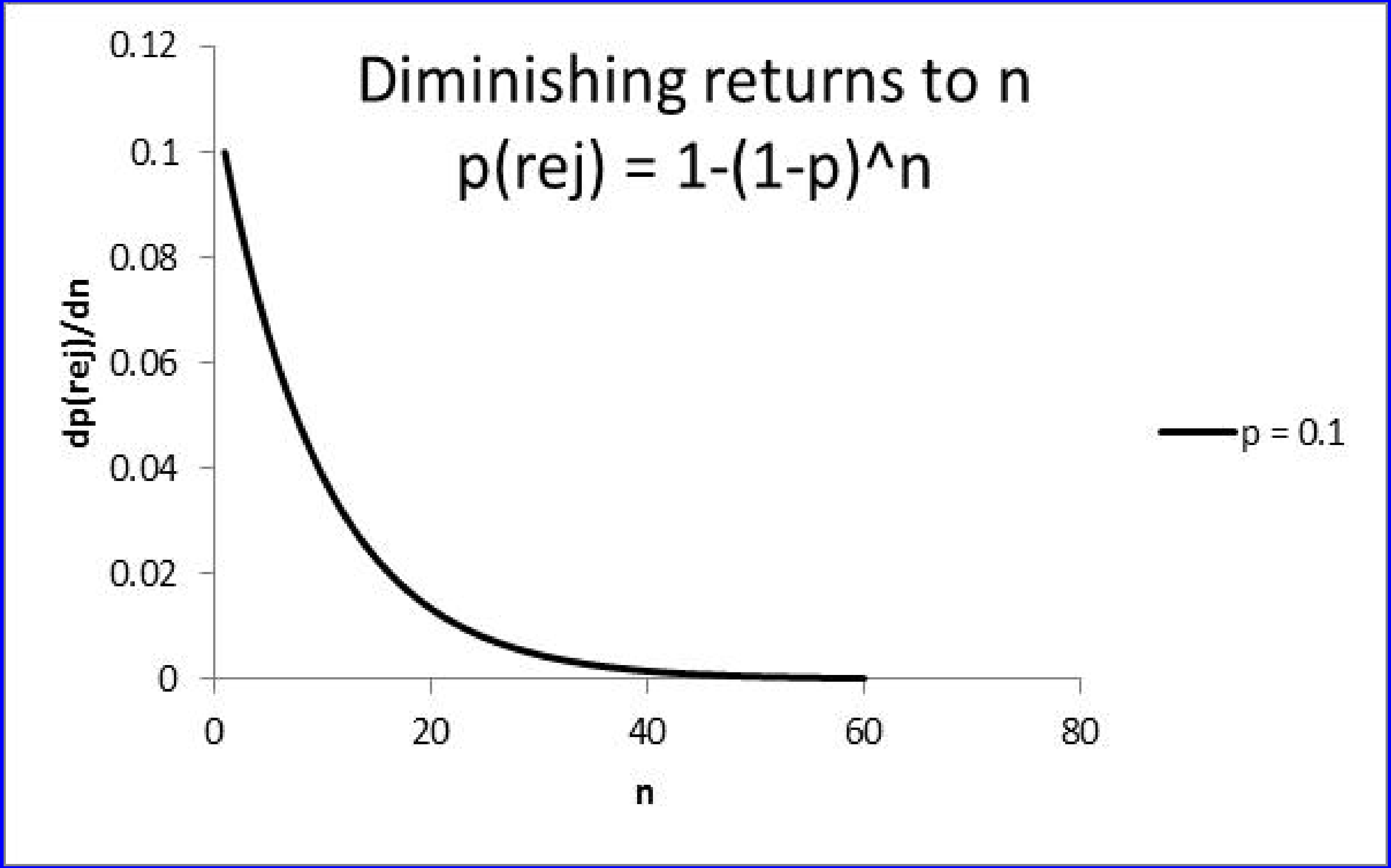
Results

- If budget constraint does not permit testing 100% of lots, n_{opt} for a given sample unit prevalence (*p*) depends only on the cost ratio (C_l/Cn).
- The budget constraint (C_T) determines absolute number of lots tested in a budget period (*m*) or the frequency of lot inspection ($1/m$)

Results


200
180
160
140
120
100
80
60
40
20
0
optimal samples per lot (nopt)
1.E-04
1.E-03
1.E-02
1.E-01
1.E+00
sample unit prevalence (p)
$Cl/$Cn
2.00
1.00
0.50
0.10
0.01
0.00

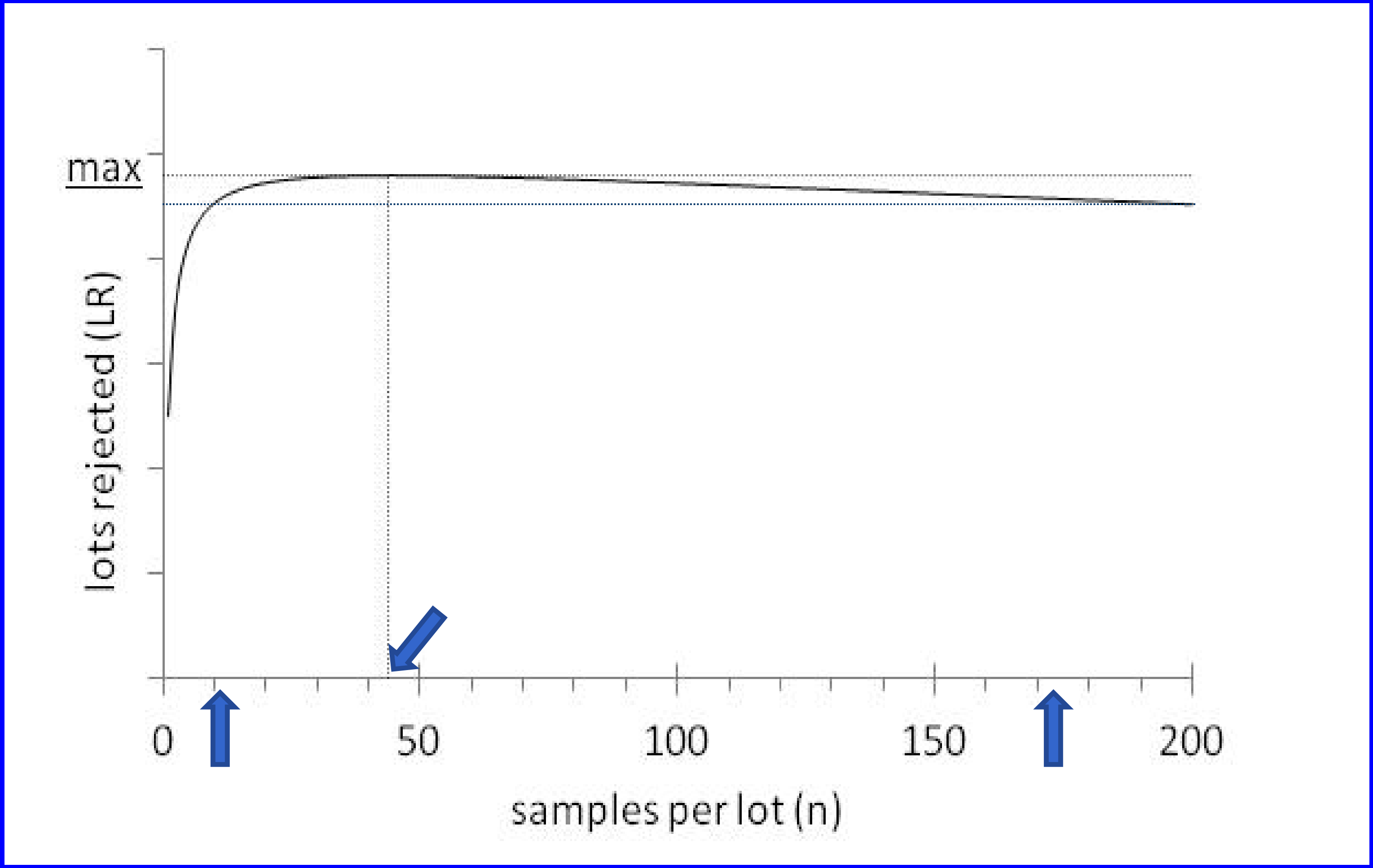
Results



Diminishing returns to n
p(rej) = 1-(1-p)^n
0.12
0.1
0.08
0.06
0.04
0.02
0
dp(rej)/dn
0
20
40
60
80
n
p = 0.1

Results

$\$C_l/\$C_n = 1$ and $p = 10^{-3}$



Conclusion

- National Research Council (1985): sampling plans based on "sound statistical concepts" need to "achieve a high degree of confidence in the acceptability of a lot."
- Economic design of measures is not new.
- Scarce resources should force us to consider the tradeoff between depth (n) and coverage (m).
- Multiple, competing objectives for sampling.

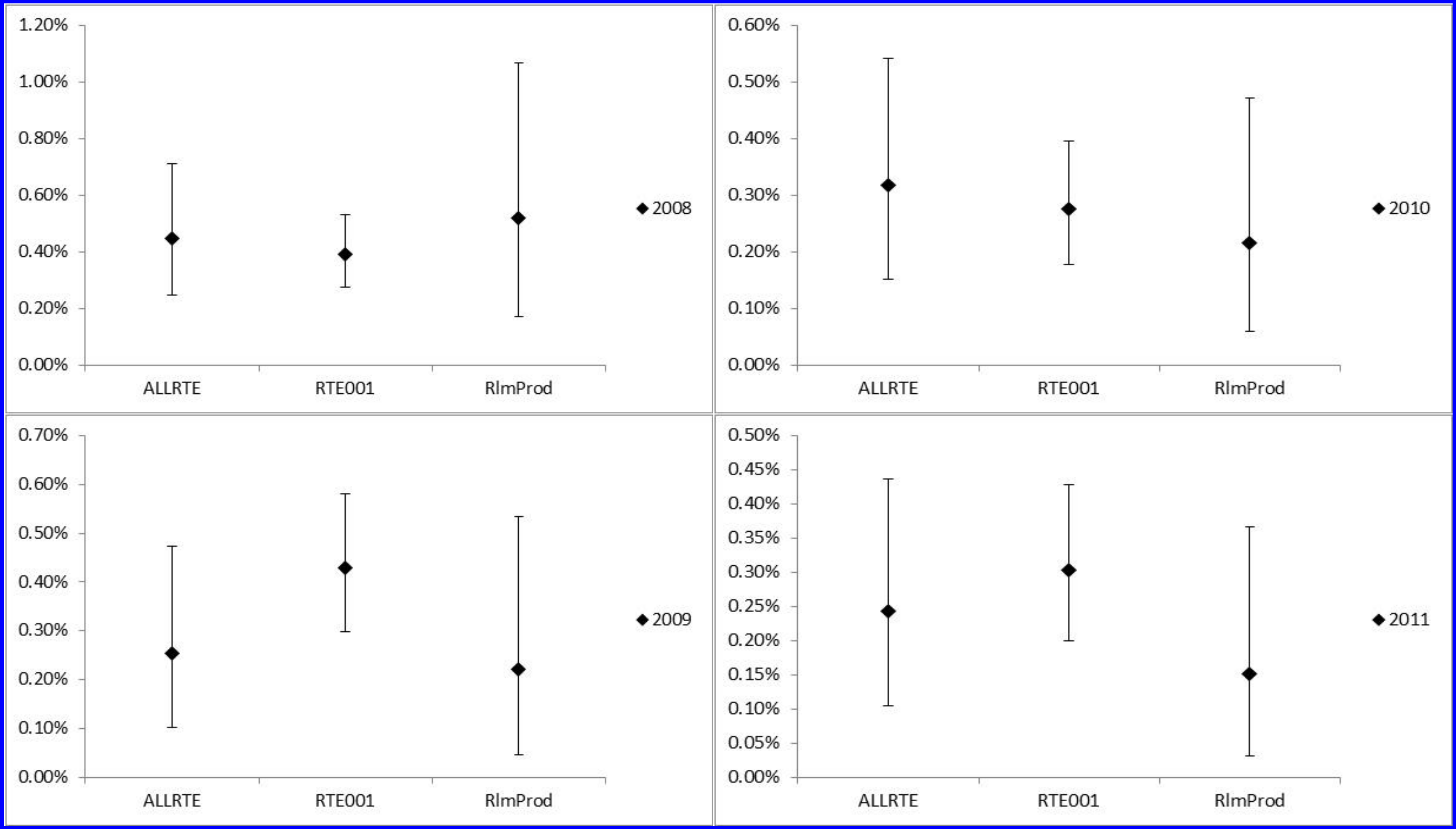
Risk-Based Sampling

– Powell, M. 2015. Risk-Based Sampling: I Don’t Want to Weight in Vain. Risk Analysis. 35(12):2172-2182.

Risk Based Inspection

- GAO (General Accounting Office) (1992). Food Safety and Quality: Uniform, Risk-based Inspection System Needed to Ensure Safe Food Supply.
- GAO (General Accounting Office) (1994). Risk-Based Inspections and Microbial Monitoring Needed for Meat and Poultry.
- USDA/OIG (U.S. Department of Agriculture, Office of Inspector General). (2007). Issues Impacting the Development of Risk-Based Inspection at Meat and Poultry Processing Establishments
- IOM (Institute of Medicine) (2009). Review of Use of Process Control Indicators in the FSIS [Food Safety and Inspection Service] Public Health Risk-Based Inspection System.
- NRC (National Research Council) (2009). Letter Report on the Development of a Model for Ranking FDA Product Categories on the Basis of Health Risks.
- NRC (National Research Council) (2009). Letter Report on the Review of the Food Safety and Inspection Service Proposed Risk-Based Approach to and Application of Public-Health Attribution.
- NRC (National Research Council) (2009). Review of the Methodology Proposed by the Food Safety and Inspection Service for Follow-up Surveillance of In-Commerce Businesses.
- NRC (National Research Council) (2009). Review of the Methodology Proposed by the Food Safety and Inspection Service for Risk-Based Surveillance of In-Commerce Activities.
- NRC (National Research Council) (2010). Enhancing Food Safety: The Role of the Food and Drug Administration.
- NRC (National Research Council) (2011). A Risk-Characterization Framework for Decision-Making at the Food and Drug Administration.

Food Safety Example: *Listeria monocytogenes* in Ready-to-Eat Meat and Poultry



Risk-Based Animal Health Surveillance

- "The rapid rate of acceptance of this core concept of risk-based surveillance has outpaced the development of its theoretical and practical bases."
  - Stark et al. 2006. "Concepts of risk-based surveillance in the field of veterinary medicine and veterinary public health: Review of current approaches." BMC Health Services Research. 6:20.

Risk Portfolio Analysis

- Prattley et al. 2007. Application of portfolio theory to risk-based allocation of surveillance resources in animal populations. Preventive Veterinary Medicine 81: 56–69.
- Cannon. 2009. Inspecting and monitoring on a restricted budget—where best to look? Preventive Veterinary Medicine 92:163–174.
- Cox. 2009. What's Wrong with Hazard-Ranking Systems? An Expository Note. Risk Analysis 29(7): 940-948.

Modern Portfolio Theory

- Markowitz (1952) Mean Variance Optimization (MVO):

$$Min_w \ \boldsymbol{w}^T \Sigma \boldsymbol{w} = \sigma_p^2$$

$$\boldsymbol{w}^T \boldsymbol{\mu} = \mu_p^*$$

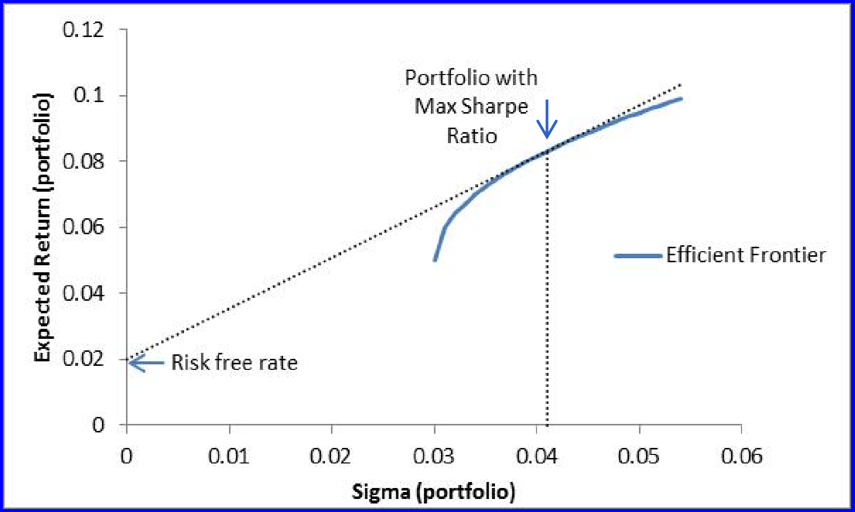
$$\boldsymbol{w}^T \mathbf{1}_N = 1$$

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

$$\mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_N \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \theta_1^2 & \cdots & \theta_{1N} \\ \vdots & \ddots & \vdots \\ \theta_{N1} & \ldots & \theta_N^2 \end{bmatrix}$$

$$\text{Sharpe Ratio (S)} = \frac{\mu_p}{\sigma_p}$$



Limitations of Mean-Variance Optimization

- Extreme asset weights
- Weights sensitive to small changes in inputs
- Poor out-of-sample performance
- Assumes stationary process
- # parameters (μ_i, σ_i^2, σ_{ij}) = 2N +(N*(N-1)/2)
- Model uncertainty

Limitations of Mean-Variance Optimization

- Sensitivity of the model to input errors
  - Frankfurter et al. 1971. "Portfolio Selection: The Effects of Uncertain Mean, Variances, and Covariances" The Journal of Financial and Quantitative Analysis 6(5): 1251-1262.
  - Hodges and Brealey. 1972. "Portfolio Selection in a Dynamic and Uncertain World" Financial Analysts Journal 28(6): 58-69.

Limitations of Mean-Variance Optimization

Example of sensitivity to estimation error

- Obj: max $S = \mu_p/\sigma_p$
- Consider two identical assets, A and B:
  - $\mu_A=\mu_B=10\%$; $\sigma_A=\sigma_B=5\%$; $\rho_{AB} = 0.9$
  - true optimal weights: $w_A = w_B = 0.5$
- Assume μ_A estimated with 10% error:
  - $\bar{x}_A = 11\%$
  - est. optimal weights: $\hat{w}_A = 0.95$; $\hat{w}_B = 0.05$

Limitations of Mean-Variance Optimization

- Naïve portfolio formation rules, such as the equal weight (1/N) rule, can outperform MVO.
  - Bloomfield, T., R. Leftwich, and J. Long. 1977. Portfolio Strategies and Performance. Journal of Financial Economics 5:201–18.

Limitations of Mean-Variance Optimization

- Very long history of returns needed to estimate mean excess return accurately
- Even if long time series are available, may not be reasonable to assume the parameters were stationary over that long a period
  - Merton. 1980. On Estimating the Expected Return on the Market. NBER Paper 444.

Limitations of Mean-Variance Optimization

- Given estimation uncertainty, optimal portfolio not well-defined; statistically equivalent portfolios with very different asset weights
  - Michaud. 1989. "The Markowitz Optimization Enigma: Is 'Optimized' Optimal?" Financial Analysts Journal. 45: 31-42.

DeMiguel et al (2007) Optimal versus Naïve Diversification

- Evaluated out of sample performance of sample-based MVO and its extensions designed to reduce effects of estimation error.
- 14 optimizing portfolio models compared to naïve diversification (1/N)
- 7 datasets of monthly returns
- Optimization models -10 year moving estimation window to predict next month's performance.
- 3 criteria: Sharpe ratio, CEQ return, and turnover

DeMiguel et al (2007) Optimal versus Naïve Diversification

- None of the portfolio optimization models performed consistently better than 1/N portfolio
- Estimation window needed for MVO and its extensions to outperform 1/N:
  - ~ 3000 months for N = 25 portfolio
  - ~ 6000 months for N = 50 portfolio

Optimal vs. Simple: Simulation

- Let $i = 1, ..., 27$ producers; $j = 1, ..., 20$ yrs.

Factor	hi	med	low
Mean lot prevalence (μ)	0.01	0.005	0.001
CV lot prevalence (σ/μ)	2	1	0.5
Volume (L_t, lots/yr)	100,000	10,000	1,000

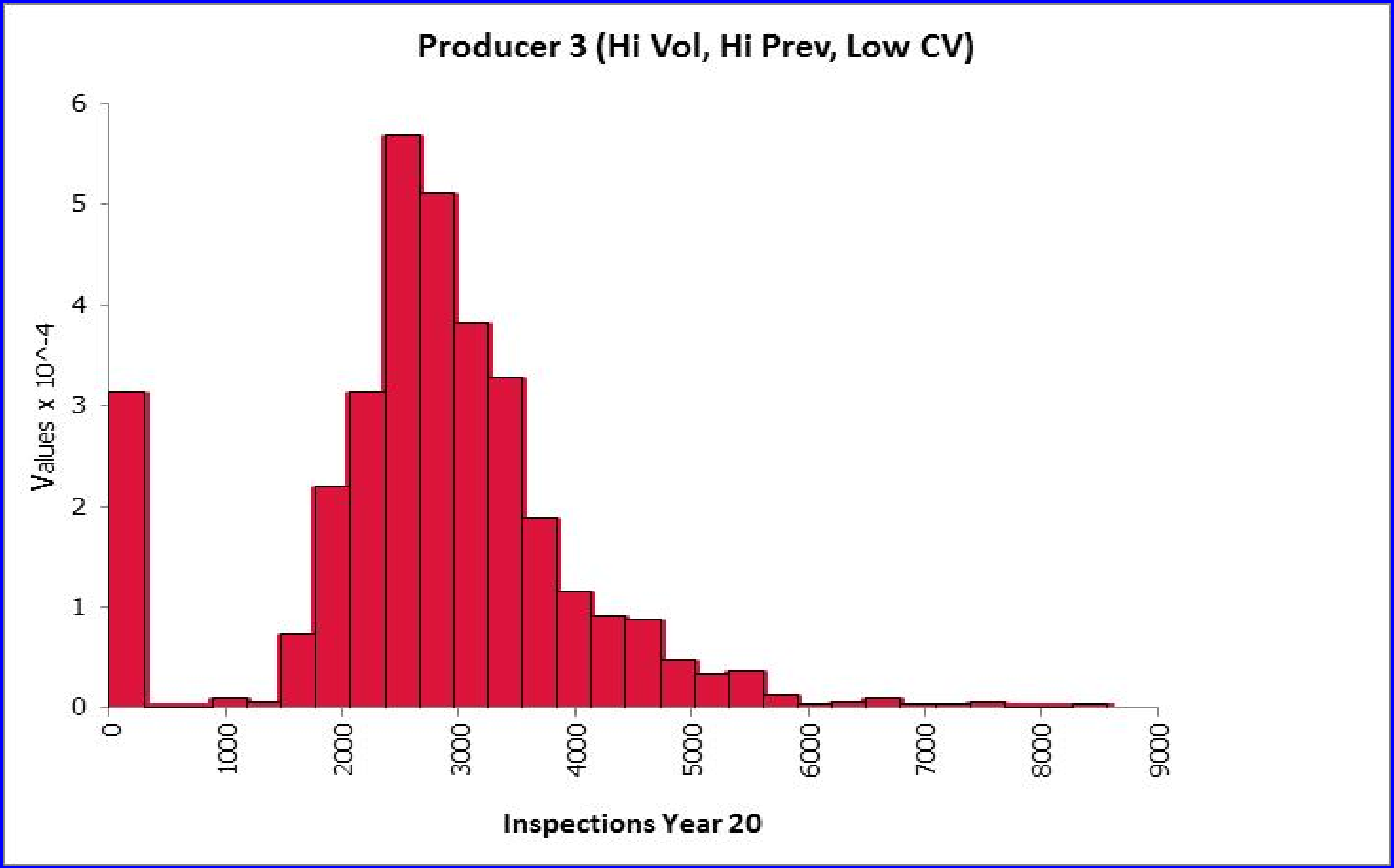
- Lot prevalence (p_i) ~ Beta(μ_i, σ_i)
- Freq. of lot inspxn ~ 1% (budget constraint)
- $\Sigma_i I_{ij}$ (lot inspxns)/yr. ~ 9,990
- p_{detxn} = 78.5% (e.g., $p_{w/in}$ = 5%, n/lot = 30)

Optimal vs. Simple: Simulation

- Simulation of Contaminated Lots & Inspection
  - #contam lots (c_{ij}) ~ Binomial(L_i, p_i)
  - #contam lot inspected (ci_{ij}) ~ Hypergeo(I_{ij}, c_{ij}, L_i)
  - #contam lots detected (x_{ij}) ~ Binomial(ci_{ij}, p_{detxn})
- Optimal Allocation: $I_{ij} \propto L_i * \hat{p}_{ij}$
  - s.t. $I_{i1} = 9990/27 = 370$; $1 \le I_{ij} \le L_i$ for j = 2, …, 20
  - $\hat{p}_{ij} = \frac{\sum_{t=1}^{j-1} x_i}{\sum_{t=1}^{j-1} I_i}$
- Simplified Allocation: $I_i \propto L_i$

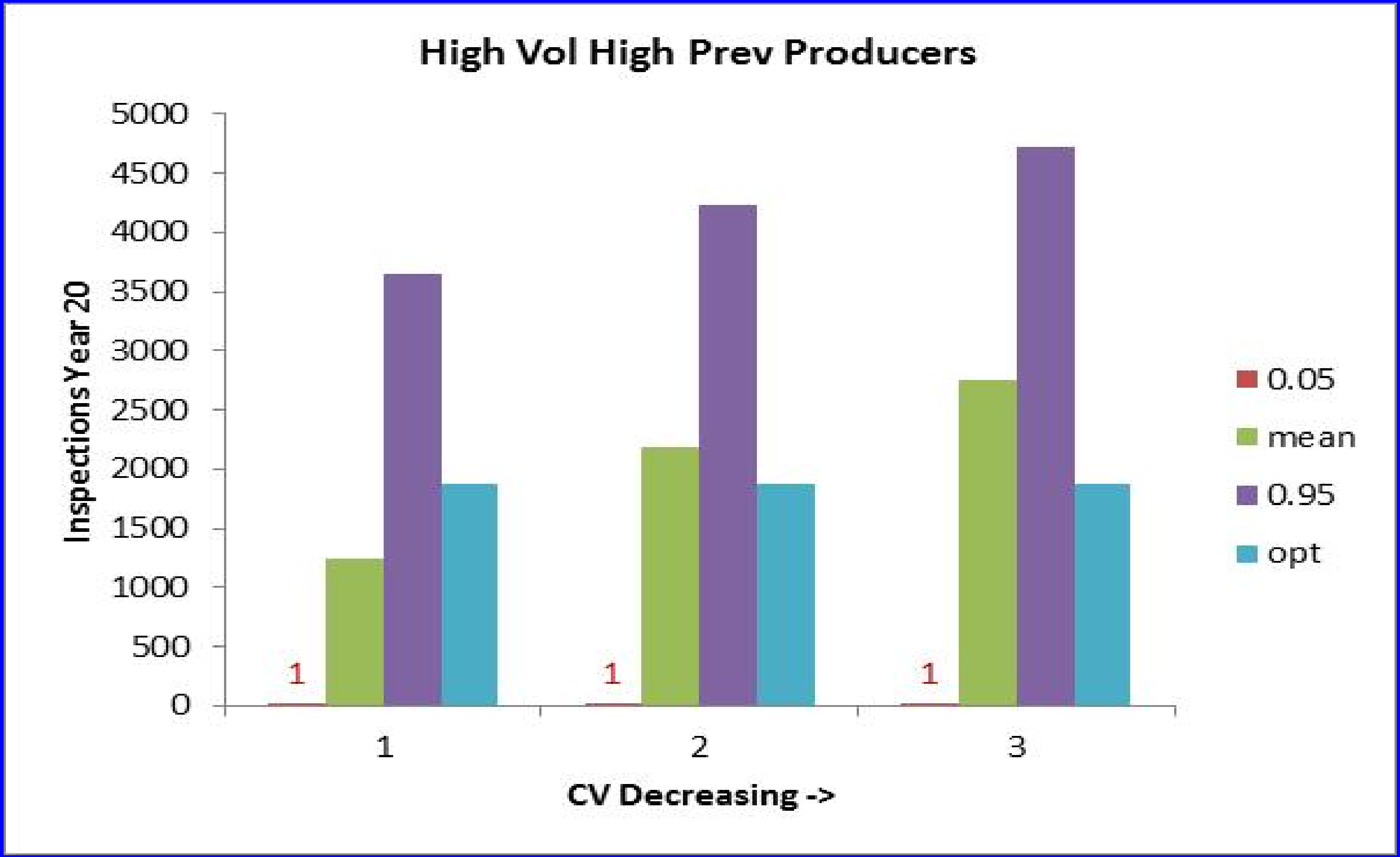
Optimal vs. Simple: Simulation

Optimized Allocation



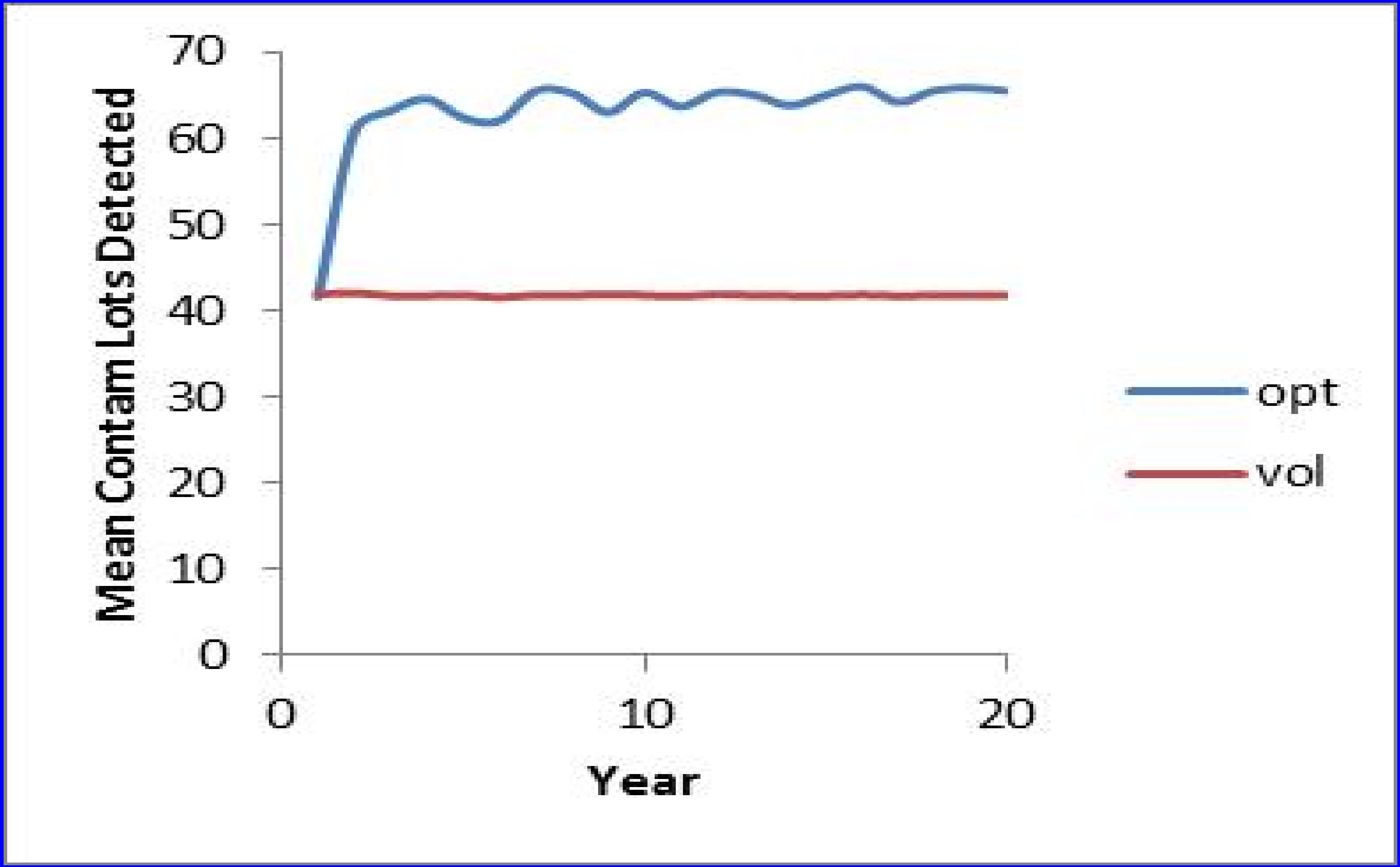
Optimal vs. Simple: Simulation

Optimized Allocation



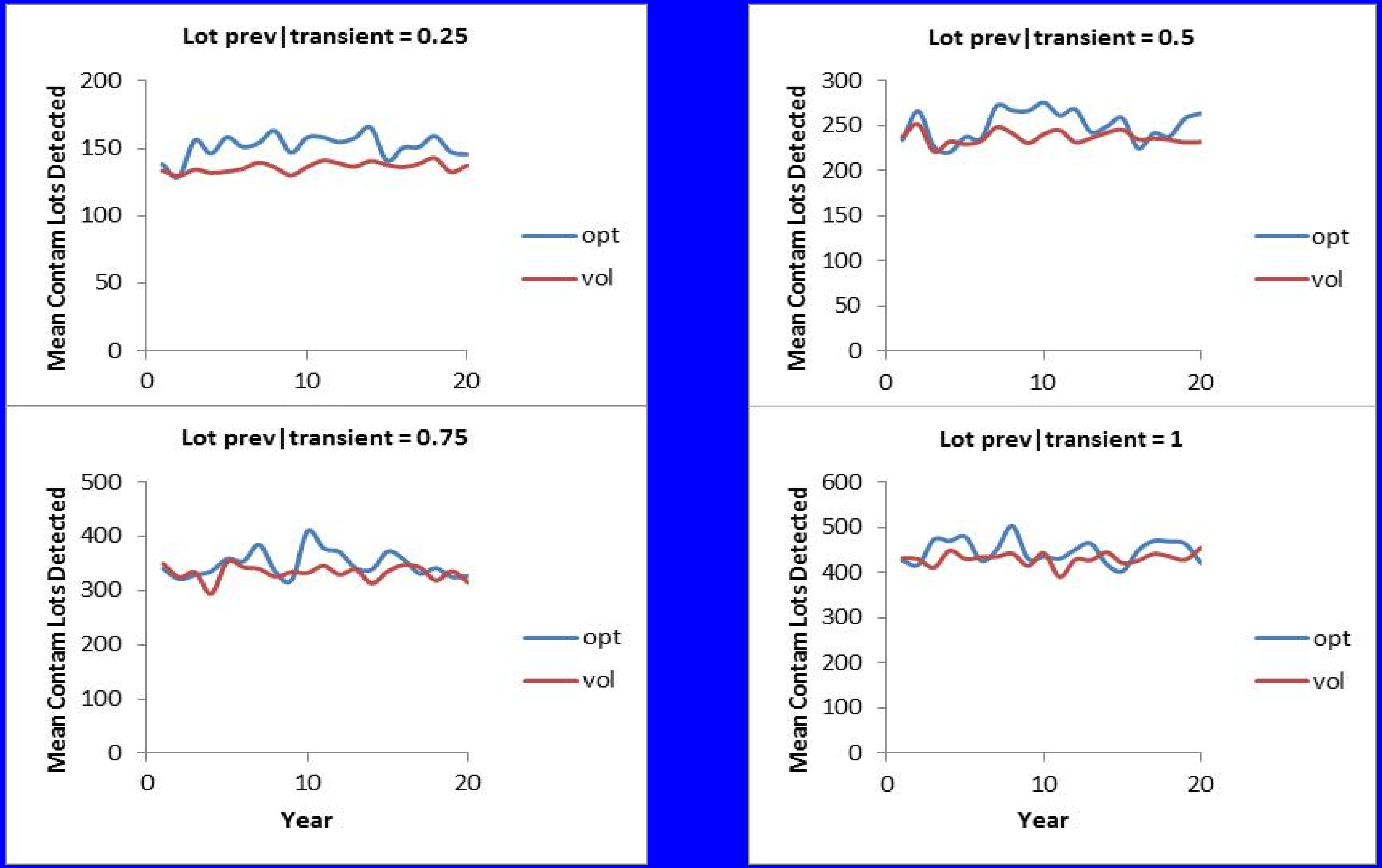
Optimal vs. Simple: Simulation

Process Stationary over 20 years



Optimal vs. Simple: Simulation

Producer's annual p(transient) = 0.05/yr. (1/20 years)



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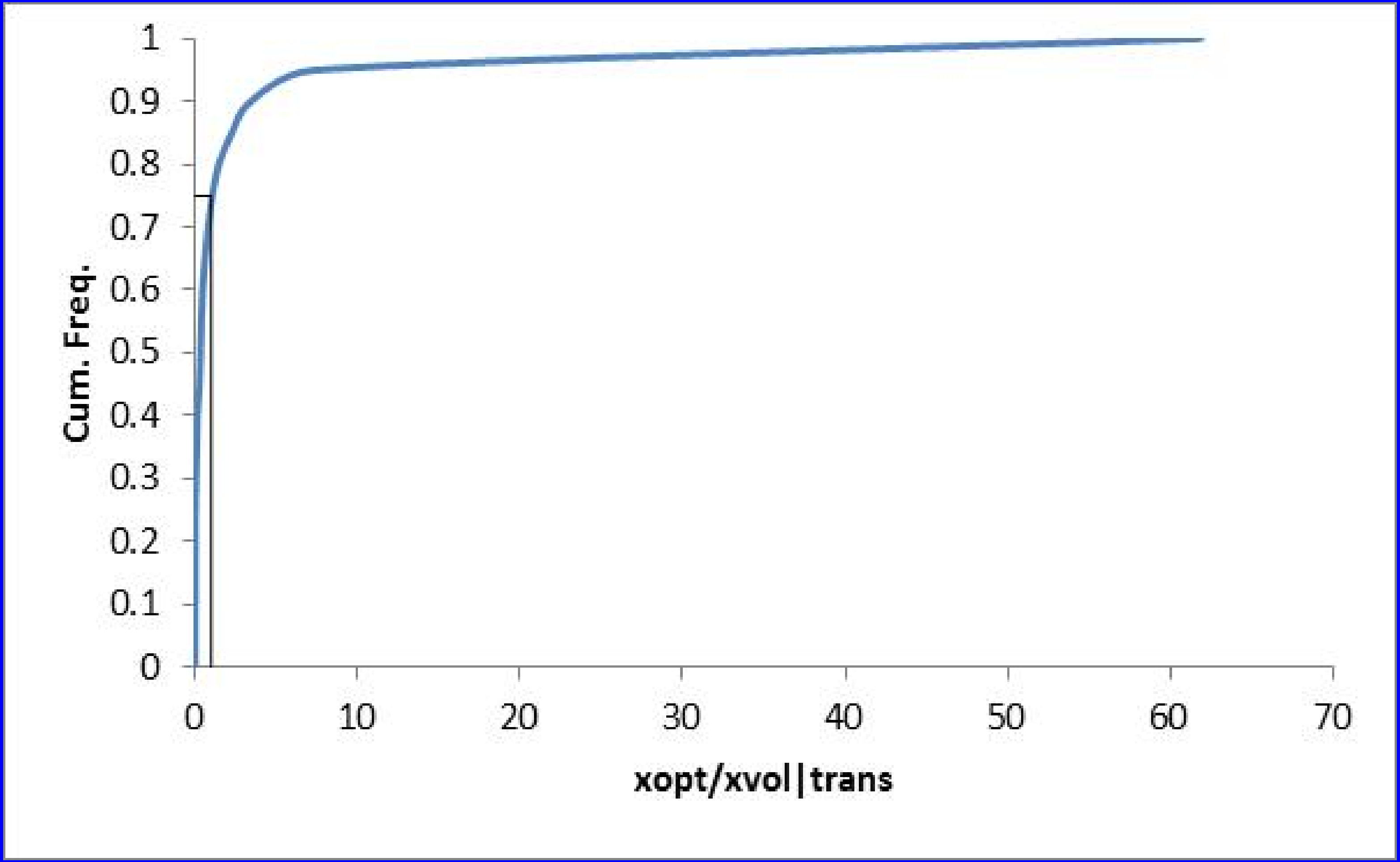

Optimal vs. Simple

- Gigerenzer. 2011. Heuristic decisionmaking. Annual Review of Psychology. 62: 451-482.
- Pflug et al. 2012. The 1/N investment strategy is optimal under high model ambiguity. Journal of Banking & Finance. 36: 410-417.
- Not all simple heuristics will outperform optimization.

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